Final Performance Report: May 1987 to August 1990

ONR Contract No. N00014-84-K-0356 (Task No. NR 042-431.)

"SPECTRAL ANALYSIS, ESTIMATION, AND PREDICTION

OF MULTIPLE HARMONIZABLE TIME SERIES"

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The following account is a summary of the work done on the continuing ONR Contract No. N00014-K-0356, by the PI and his associates during the three year period indicated above.

A large part of the proposed research in the contract has been accomplished. This is reflected in the completion and publication of six research papers, editing of a monograph, and preparation of a graduate level text-research type volume on the Theory of Orlicz spaces which will be useful for applications later. Also, partial supports are extended to two graduate students who have completed their Ph.D. theses during this period, and to two visiting researchers. Details of this work and related activities are as follows.

1. Research Papers Prepared.

(a) The problems of moving average representations of weakly harmonizable processes, extending the corresponding results on stationary processes, is of interest both-theoretically and for applications. This was studied by the PI in collaboration with Derek K. Chang, the results of which were presented in the journal article:

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"Special representations of weakly harmonizable processes," Stoch. Anal. Appl., 6(1988), 169-189.

In this paper, series as well as operator representations, and relations between oscillatory and periodically correlated processes, in the context of harmonizable classes are considered. The work of this paper has led to further analysis of strongly harmonizable processes which constituted a Ph.D. thesis of a graduate student. (See 4 (ii) below.)

(b) Completing some earlier work on the computational problems of conditional probabilities, some unresolved questions are highlighted with illustrations in the paper:

"Paradoxes in conditional probability," J. Multivariate Anal., 27(1988), 434-446.

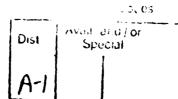
(c) If $\Lambda X_t = Y_t$, $t \in \mathbb{R}$, is a general filter equation where Λ is a (constant coefficient) difference, differential or integral operator and the Y_t is a stationary or harmonizable process called the "output", then one needs to find conditions on Λ in order that a harmonizable or stationary "input" solution X_t may exist. This problem under various conditions was treated in the paper:

"Harmonizable signal extraction, filtering, and sampling," in *Topics in Non-Gaussian Processing* (Eds. E.J. Wegman, S.C. Schwartz, and J.B. Thomas) Springer-Verlag, New York, (1989), 98-117.

After a long delay of this publication, the paper appeared with slight modifications. It includes a sharper version of an early theorem by S. Bochner, together with several related results. In particular, physical realizability, and sampling problems are also discussed.

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(d) To a large extent, the theory of harmonizable processes depends on the properties of bimeasures which appear as spectral measures of these classes. This is needed even for classification and prediction problems. The relevant analysis from a general view-point including the case of multiple index [i.e. random fields] is discussed in considerable detail in the paper:

"Bimeasures and harmonizable processes, (analysis, classification, and representation)," Proc. Oberwolfach conference on *Probability Measures on Groups*, (Ed. H. Heyer), Springer Lact. Notes in Math., #13-79 (1989), 254-298.

The work here will form a basis for much of the ensuring research on harmonizable processes and fields, the index being allowed to be certain topological groups. Also, included is the work on strict harmonizability, related to strictly stationary and stable processes. All these classes are important for potential research areas with substantive applications.

(e) An analysis of the structure and spectral domain of strongly harmonizable processes, including a result on asymptotically unbiased estimators of the spectral distribution based on one realization of the process, are given in the paper:

"A view of harmonizable processes," Proc. of Conference on Statistical Data Analysis and Inference (ed. Y. Dodge), North-Holland Publishers (1989), 597-615.

This paper contains a discussion on problems of inference for harmonizable processes of low multiplicity and also conditions for the strong law of large numbers.

(f) Thus far, most of the work discussed in detail is on processes, i.e. the indexing of X_t is either the reals \mathbb{R} or the integers \mathbb{Z} . The corresponding problems when the

index is \mathbb{R}^n [or \mathbb{Z}^n], n > 1, admit further analysis for new properties, namely that of isotropy. Considerable amount of work for stationary (= homogeneous) isotropic random fields is available, most of it being done in the USSR. The corresponding new concept of harmonizable isotropy is recently introduced and a representation of these random fields has been obtained. Problems of sampling and prediction are then considered. Some of this work will appear in the paper:

"Sampling and prediction of harmonizable isotropic random fields," J. Combinatorics,

Information & System Sciences (1990/91).

The concept here will be useful in the analysis of several problems. The original homogeneous isotropic fields were treated especially by M.I. Yadrenko and A.M. Yagloun. These are being studied for the general harmonizable case in our research.

(g) An updated and expanded version of the article entitled "Probability" is presented for inclusion in the Second Edition of the Encyclopedia of Physical Science and Technology, (Vo. 11), Academic Press, which is scheduled to appear in 1991. This enlarged version of the PI's original article of 1986, includes a discussion of harmonizable processes as a new item from the earlier paper.

2. Books Completed.

(a) The following research monograph is edited jointly by the PI and Prof. C.R. Rao of the Penn State University:

Multivariate Statistics and Probability, Academic Press Inc., New York, (1989), 567 pages.

(b) The following is a graduate level text-reference work, completed by the PI jointly with Z. D. Ren, visiting UCR, and is expected to be published by the end of 1990 or soon thereafter:

Theory of Orlicz spaces, Marcel Dekker Inc., New York, (approx. 450 pages).

3. Participation in Professional Meetings.

- (i) The PI is one of the five special invited speakers for the five day conference on *Probability Measures on Groups*, held in January 1988 at Oberwolfach, West Germany. The presented paper entitled "Bimeasures and harmonizable processes", is prepared in an expanded form, including the substance of the talk. It appeared later as the paper listed as 1 (d) above.
- (ii) The PI was invited to present a talk at the conference on Statistical Data Analysis and Inference held at the Université de Neuchâtel, in Switzerland, in August 1989. The trip was partly supported by the ONR contract. The complete version of this talk has appeared as paper 1 (e) described above.
- (iii) The PI was also invited to participate in the "Analysis" conference held at Pozńan, Poland in 1987, but he could not go.
- (iv) The PI gave two talks in Mathematics Seminar, in August 1987, at the Central University, Hyderabad, A.P., India.
- 4. Graduate Student Support and Training.

During the period under consideration, partial support from the contract is extended

for graduate study to two students who have now completed their Ph.D. degrees at UCR, and later are employed. They are:

(i) Joseph J. Sroka, completed in December 1987, with a dissertation entitled:

"PNA games through infinite dimensional Banach spaces". [Sroka is now employed at the University of Scranton, Scranton, PA.]

(ii) Marc H. Mehlman, completed in March 1990, with a dissertation entitled:

"Moving average representation and prediction for multidimensional strongly harmonizable processes". [Mehlman is now employed at the University of Pittsburgh at Johnstown, Johnstown, PA.]

- (iii) Another graduate student (Randall J. Swift) has just started his work for a thesis in the area of harmonizable processes, and a partial support is provided to him during the last part of the contract period.
- 5. Research Scholars Visiting the PI at UCR.
- (i) Dr. M. Tsukada of the Science University of Tokyo, Japan, has recently spent the year from April 1989 to March 1990 as a post doctoral fellow. He came with a fellowship from his own institution. He has returned to his university.
- (ii) During 1988, Derek K. Chang, from California State University at Los Angeles, spent some time here and worked with the PI, and this resulted in the completion of the joint paper listed as 1 (a) above. His participation was noted in the ONR proposal, and he was thus supported for the summer time. Later on he did not continue to come or